

A Novel Method to Translate a Physical Watershed into Matrix Form for Run-off Estimation

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Abstract—This article presents innovative notions of translating a micro-watershed into a unique matrix using a graphical representation of a tree graph in graph theory assigning the order of the streams to the array of each node. It addresses new concepts of basic constituent, base and characteristic matrices of a micro-watershed. The main objective behind translating a watershed into a matrix is to develop a sediment transport model concerning different attributes of drainage system and topographical area of each micro-watershed in a river basin to predict water flow and quantify sedimentation to the reservoir through simulation. These innovations promise to advance our understanding of watershed dynamics and provide us a future scope to develop a sediment prediction which is crucial for water resource management and environmental planning. This study focuses on estimating the water flow from the contributing watersheds of the Barakar River Basin. A mathematical modeling and computer simulation of run-off from contributory topographical areas has been carried out to simulate water flow incorporating the stream order, contributory area, run-off and rainfall as matrices using the graph theory based algorithm. Key input parameters such as land use, land cover changes, soil properties, and topographical features are derived from the satellite images processed from Remote sensing techniques to incorporate their influence on water flow dynamics. The findings provide valuable insights into the hydrological behaviour of the watersheds that leads to forecast and control the flood and also aids to devise sustainable water resource management strategies in Barakar River Basin as well as water conservation and management in Maithan reservoir.

Keywords: watershed matrix, graph theory, sediment transport, easy data handling and memory management, run-off estimation

INTRODUCTION

Hossain (2015) studied and highlighted that water plays a crucial role in religious rituals. However, its distribution across the globe varies seasonally and by location. Throughout history, dams and reservoirs have been pivotal in collecting and managing water to support civilizations, as highlighted by Dwivedi *et al.* (2010). India, for instance, relies heavily on its 4500 large reservoirs and numerous smaller ones, storing water from the monsoon season to sustain non-monsoon periods. Siltation poses a significant challenge, with studies showing a loss of live storage

capacity in many reservoirs, estimated at 214.2 million cubic meters annually (CWC, 2009) that costs upto a substantial loss of approximately Rs. 5.53 crores daily. Sedimentation threatens river ecosystems globally, with research indicating a decreasing flow trend in 50% of major rivers due to sediment accumulation (Walling & Fang, 2003). Globally, reservoirs are experiencing an annual sedimentation rate of about 31 cubic kilometers, potentially halving their storage capacity by 2100 (Sumi & Hirose, 2009). To mitigate these challenges, innovative approaches using geomorphological

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data and river flow simulation models are being developed to predict sediment deposition, manage reservoir capacity, and optimize downstream water discharge.

MATERIALS AND METHODS

STUDY AREA

The chosen research area is the Maithon Reservoir, offering advantages such as hydroelectric power generation with a 60 MW capacity from three 20 MW units located underground on the left bank, supplying water to Dhanbad district, and serving as a popular tourist destination.

SOME INNOVATIONS

Understanding basic geomorphological concepts of a river basin is crucial before developing a reservoir model. A micro-watershed, the fundamental unit, is defined as an area bounded by natural boundaries where runoff drains through a common outlet. Watersheds are unions of micro-watersheds, and river basins are unions of watersheds, represented using graph theory and matrix algebra principles.

BASIC CONSTITUENT OF A DRAINAGE SYSTEM

Exploring innovative concepts for drainage system representation involves understanding geomorphological features within micro-watersheds. Nodes collect runoff, and edges channel it, combining local sources into significant tributaries. Initial designs focus on main models, later integrating point sources using statistical methods from geomorphological studies. Connectors gather runoff where stretches meet, while receivers mark downstream terminations, forming a structured tree graph with collectors and transporters in the drainage system as shown in Fig.1

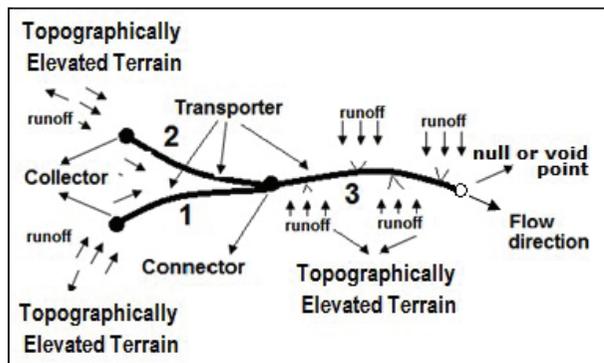


Fig. 1: An Illustrative Drainage System Showing Three Basic Constituents.

The Barakar River basin is mapped using GIS and Remote Sensing, and algorithmically coded to convert it into matrix form, involving detailed observation of each river section (Fig 2).

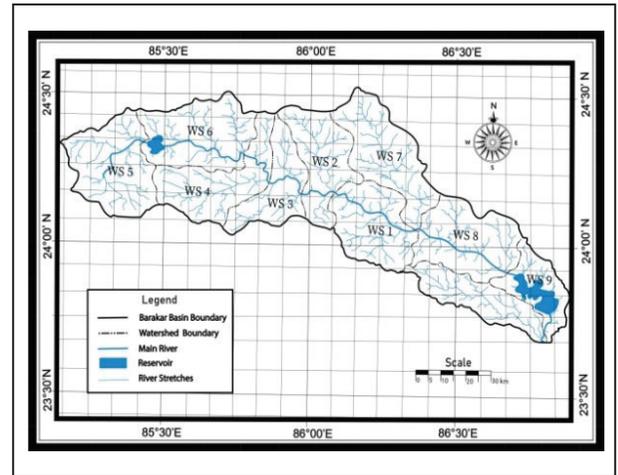


Fig. 2: Delineated Map of the Barakar River Basin.

ESTIMATION OF RUN-OFF USING ALGORITHM

An algorithm for converting a micro-watershed into a matrix is as mentioned below. The watersheds shown in Fig.2 are individually transformed into the order matrices by following the steps as follow:

1. Name nodes of stretch order 1 from the left side of the main river near the final outlet, moving clockwise as $A_{1,1}, A_{1,2}, A_{1,3}, \dots, A_{1,n}$ where the first subscript denotes stretch order and the second denotes serial number or micro-watershed order in clockwise direction.
2. Repeat for stretch order 2 nodes as $A_{2,p}$ where $n \leq p \leq 1$ and p is the serial number of the first water flow contributor clockwise from stretch order 1, as shown in Fig. 2.
3. Continue similarly for stretch order 3 nodes as $A_{3,q}$ where $n \leq q \leq 1$ and q is the serial number of the first water flow contributor clockwise from stretch order 1, as depicted in Fig. 4, repeating until naming the last node of maximum order n as $A_{n,r}$.
4. The value of the element $A_{i,j}$ is defined for the node whose latitude and longitudes is (x,y,z) and the order is j as follows:

$$A_{i,j} = \begin{cases} (x,y,z,i) & \text{if node P exists corresponding to the value j} \\ 0 & \text{otherwise} \end{cases}$$

The ordered 4-tuple (x, y, z, i) is necessary only during the computation of various measures involved in the reservoir modeling. This order matrix, derived from the micro-watershed, serves as a base matrix, with other attribute matrices derived from it for comprehensive drainage system representation. The Barakar River basin is mapped into 9 watersheds, further divided into micro-watersheds for individual order matrix conversion. The study utilizes five-year average rainfall data (2016-2020) from Dhanbad, Giridih, Hazaribagh, and Koderma

districts, provided by JSAC, Jharkhand. Runoff calculation involves **Total Run-off (TR) = Contributory Area (CA) × Rainfall (R)**, adjusted for infiltration and evaporation at approximately 30% in the basin, determining the Final Estimated Quantity of Run-off (Q) into the river basin as **Q = TR - Infiltration rate (%) × TR**.

RESULTS

The order matrices of the micro-watersheds that fall under the watershed WS1 as shown in Fig.2 are illustrated below:

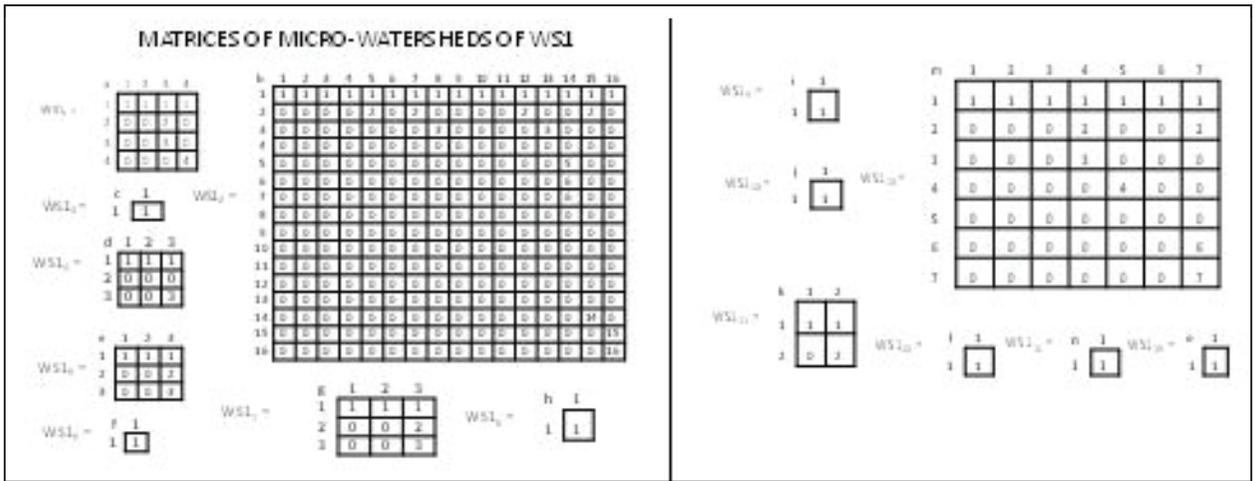


Fig. 3: Representation of 15 Micro Watersheds of WS1 into 15 Order Matrices.

All nine watersheds are transformed into matrix form to derive characteristic matrices based on contributory area, slope, elevation, stretch length, and rainfall, with Table 1 detailing run-off quantity (Q) from WS1 with infiltration deduction. Also, we have shown below the graph of the total run-off from all watersheds as shown in Fig. 4

Table 1: Represents Quantity of Run-off from WS1 into the Barakar River Basin.

Month	Total Rainfall Monthwise in Millimetre	Total Quantity of Run-off in Cubic Metre
Jan	6.96	3054223.81
Feb	10.18	4467241.147
Mar	18.4	8074384.784
Apr	30.46	13366617.42
May	62.6	27470461.28
Jun	130.96	57468556.05
Jul	305.88	134227870.5
Aug	239.86	105256626.9
Sep	236.52	103790950.5
Oct	64.4	28260346.74
Nov	21	9215330.46
Dec	6.76	2966458.758
Total	1133.98	497619068.3

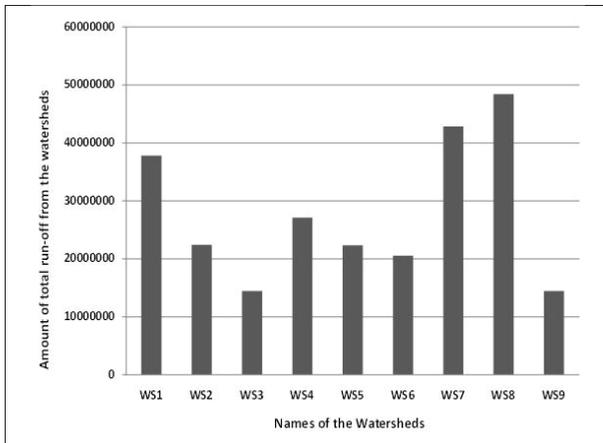


Fig. 4: Graph of the Total Contribution of Run-off by the Nine Watersheds (WS1 to WS9) into the River Basin.

DISCUSSIONS

During the rainy months of June, July, and August, increased rainfall in watersheds leads to higher runoff into the reservoir. Factors such as slope, elevation, and velocity affect runoff; higher slopes typically result in greater velocities and increased runoff. It's crucial to compute and store flow rates for each micro-watershed stretch in matrix form to predict sediment settlement in river beds and particles flushed into the reservoir. Flow rate is calculated as:

$$\text{Flow rate} = \text{Stream Velocity} \times \text{Contributory Area.}$$

CONCLUSION

Our approach integrates micro-watershed nodes with various river attributes, optimizing water flow predictions and flood control. The algorithm enhances data handling efficiency and computational speed, crucial for accurate sediment and runoff predictions across diverse landscapes. Integrating infiltration rates into runoff calculations ensures precise reservoir sediment forecasts, crucial for water management strategies.

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