

3D Analysis of Wave Propagation in Generalized Magneto Thermoelastic Medium

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Abstract—The paper investigates the three dimensional problem of wave propagation in generalized magneto thermoelastic medium. The eigenvalue approach is adopted to solve the vector matrix differential equation which represents the equation of motion for the generalized thermoelastic materials. Components of displacements and temperature field are analyzed theoretically and computed numerically with the help of appropriate boundary conditions. The effect of magnetic and relaxation time on the propagation of waves are critically examined.

Keywords: Generalized magneto thermoelastic material; eigenvalue; Heaviside function; Phase velocity; wavenumber.

INTRODUCTION

The theory of thermoelasticity has many applications in various fields. Biot (1956) investigated the first paradox relating to the decoupling of deformation field of an elastic material with the temperature field. The contradiction of the infinite speed of the thermal wave was solved by Lord and Shulman (1967) in their generalized thermoelastic theory. Dhaliwal and Sherief (1980) derived the variation principle and uniqueness theorem for the generalized anisotropic thermoelastic material. Several authors, such as Puri (1973), Paria (1966) and Ezzat and Youssef (2005) contributed to the development of generalized thermoelastic theory. Youssef (2005) studied the generalized thermoelasticity of an infinite body with a cylindrical cavity and variable material properties. Wang and Dai (2004) introduced a theoretical method to analyse the magnetothermoelastic responses and perturbation of the magnetic field in the orthotropic thermoelastic cylinder subjected to thermal shock.

Zorammuana and Singh (2016) investigated the problem of elastic wave at a plane free boundary of thermoelastic saturated porous half-space and obtained the amplitude and energy ratios of the reflected waves. Abbas (2012) discussed the transient phenomena in the magneto-thermoelastic model in the context of the Lord and Shulman theory in a perfectly conducting medium. Some interesting problems of wave and vibration can be seen in (Abbas, 2013; Abbas *et al.*, 2012; Abbas and Othman, 2012 and Dhaliwal *et al.*, 1980).

The analysis of three dimensional thermoelastic problem has been found relatively less attention due to complicated form of the solution. Here, in this paper, we obtain the components of displacement theoretically and numerically using eigenvalue approaches for the generalized magneto thermoelastic medium. The equation of motion for such medium are derived using Maxwell's equations and modified Ohm's law. The numerical results are plotted to see the effects of thermal relaxation time and magnetic field.

3D Analysis of Wave Propagation

Table 1: Nomenclature.

| Symbols | | Symbols | |
|----------------|-------------------------------|---------------|-----------------------------|
| λ, μ | Lamé constants | ρ | density |
| C_e | specific heat | t | time |
| T | absolute temperature | T_0 | reference temperature |
| e_{ij} | strain tensor | \mathbf{u} | displacement vector |
| k_0 | thermal conductivity | \mathbf{J} | current density vector |
| μ_0 | magnetic permeability | σ_{ij} | stress tensor |
| τ | relaxation time | e | cubical dilatation |
| α_T | linear thermal expansion | \mathbf{H} | magnetic intensity |
| H_0 | initial magnetic field | \mathbf{D} | electric displacement |
| \mathbf{E} | induced electric field | \mathbf{h} | induced magnetic field |
| σ_0 | electric conductivity | \mathbf{B} | magnetic inductance vector |
| δ_{ij} | Kronecker's delta | γ | $(3\lambda + 2\mu)\alpha_T$ |
| c_1 | $\sqrt{\mu/\rho}$ | c_2 | $\sqrt{\mu_0 H_0^2/\rho}$ |
| c_3 | $\sqrt{(\mu + \lambda)/\rho}$ | c_4 | $\sqrt{\gamma/\rho}$ |

BASIC EQUATIONS

The equation of motion in the absence of body force and heat source for linearly homogeneous magneto thermoelastic materials are

$$\sigma_{ij,j} + (\mathbf{J} \times \mathbf{B})_i = \rho \ddot{u}_i \quad (1)$$

The Maxwell's equations for homogeneous, thermally and electrically conducting elastic solid are given by

$$\text{curl } \mathbf{H} = \mathbf{J}, \quad \text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \text{div } \mathbf{B} = 0, \quad \mathbf{B} = \mu_0 \mathbf{H}. \quad (2)$$

The modified Ohm's law for finite conductivity is

$$\mathbf{J} = \sigma \left[\mathbf{E} + \left(\frac{\partial \mathbf{u}}{\partial t} \times \mathbf{B} \right) \right]. \quad (3)$$

If $\mathbf{H} = (H_x, H_y, 0)$, $\mathbf{E} = (E_x, E_y, 0)$ and $\mathbf{u} = (u, v, w)$ in Eqs.(2) and (3), we get

$$-\mu_0 \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \quad \mu_0 \frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}, \quad (4)$$

where

$$E_x = \frac{-1}{\sigma} \frac{\partial H_y}{\partial z} + \mu_0 H_y \frac{\partial w}{\partial t}, \quad E_y = \frac{1}{\sigma} \frac{\partial H_x}{\partial z} - H_x \mu_0 \frac{\partial w}{\partial t},$$

$$E_z = \frac{1}{\sigma} \frac{\partial H_y}{\partial x} - \frac{1}{\sigma} \frac{\partial H_x}{\partial y} + \mu_0 H_x \frac{\partial v}{\partial t} - \mu_0 H_y \frac{\partial u}{\partial t} - \mu_0 \frac{\partial H_y}{\partial t}.$$

Thus, the components of \mathbf{J} are given by

$$J_x = \sigma \left(E_x - \mu_0 H_y \frac{\partial w}{\partial t} \right), \quad J_y = \sigma \left(E_y + \mu_0 H_x \frac{\partial w}{\partial t} \right),$$

$$J_z = \sigma \left(\mu_0 H_y \frac{\partial u}{\partial t} - \mu_0 H_x \frac{\partial v}{\partial t} + E_z \right).$$

Equation(4) is linearized by inserting $H_x = H_0 + h_x$ and $H_y = H_0 + h_y$ so that H_x and H_y denote change in the basic magnetic field, H_0 along the x and y directions respectively. These changes can be obtained using Eq.(4) as

$$h_x = -H_0 \left(\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y} + \frac{\partial w}{\partial z} \right), \quad h_y = -H_0 \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} + \frac{\partial w}{\partial z} \right). \quad (5)$$

The constitutive relations of generalized thermoelastic solid are given by

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma \delta_{ij} \Psi, \quad \bar{\sigma}_{ij} = H_i h_j + H_j h_i - (\mathbf{H} \cdot \mathbf{h}) \delta_{ij}. \quad (6)$$

where,

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}, \quad \psi = T - T_0.$$

The stress tensors are given by

$$\begin{aligned} \sigma_{xx} &= (\lambda + \mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \gamma \psi, \\ \sigma_{yy} &= \lambda \frac{\partial u}{\partial x} + (\lambda + \mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial w}{\partial z} - \gamma \psi, \\ \sigma_{zz} &= \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right), \quad \sigma_{zz} = \lambda \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} + (\lambda + \mu) \frac{\partial w}{\partial z} - \gamma \psi, \\ \sigma_{yx} &= \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad \sigma_{zy} = \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right). \end{aligned}$$

Thus, the equations of motion for homogeneous magneto thermoelastic materials are derived using Eqs.(1) and (6) as

$$(c_3^2 - c_2^2)u_{xx} + (c_1^2 + c_2^2)u_{yy} + c_1^2u_{zz} + c_2^2(v_{xx} - v_{yy}) + c_3^2v_{yx} + (c_3^2 - c_2^2)w_{xz} - c_2^2w_{zy} - c_4^2\psi_x = u_{tt}, \quad (7)$$

$$c_2^2(u_{xx} - u_{yy}) + (c_1^2 - c_2^2)v_{xx} + (c_2^2 + c_3^2)v_{yy} + c_3^2u_{xy} + c_1^2v_{zz} + (c_2^2 + c_3^2)w_{yz} - c_4^2\psi_y = v_{tt}, \quad (8)$$

$$c_1^2(w_{xx} + w_{yy}) + (c_3^2 + 2c_2^2)w_{zz} + (c_2^2 + c_3^2)u_{xz} - c_2^2u_{yz} + (c_2^2 + c_3^2)v_{yz} - c_4^2\psi_z = w_{tt}, \quad (9)$$

$$k_0\psi_{ii} = (\psi_{,t} + \tau\psi_{,tt}) + \gamma T_0(e_{,t} + \tau e_{,tt}). \quad (10)$$

THREE DIMENSIONAL SOLUTION

We assume that the solutions of Eqs.(7)-(10) take the following form

$$\{u, v, w, \psi\}(x, y, z, t) = \{\bar{u}, \bar{v}, \bar{w}, \bar{\psi}\}(z)e^{i(\alpha x + \beta y - \omega t)}, \quad (11)$$

where α, β are wave numbers along x and y directions respectively and ω is angular frequency.

Using Eq.(11) into the equations of motions (7)-(10), we have the following set of equations

$$\frac{d\bar{V}}{dz} = A\bar{V}, \quad (12)$$

where

$$\bar{V} = \left[\bar{u}, \bar{v}, \bar{w}, \bar{\psi}, \frac{d\bar{u}}{dz}, \frac{d\bar{v}}{dz}, \frac{d\bar{w}}{dz}, \frac{d\bar{\psi}}{dz} \right]^T,$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ a_{51} & a_{52} & 0 & a_{54} & 0 & 0 & 0 & a_{57} \\ a_{61} & a_{62} & 0 & a_{64} & 0 & 0 & a_{67} & 0 \\ 0 & 0 & a_{73} & 0 & a_{75} & a_{76} & 0 & a_{78} \\ a_{81} & a_{82} & 0 & a_{84} & 0 & 0 & a_{87} & 0 \end{bmatrix}.$$

$$a_{51} = (c_{31}^2 - c_{21}^2)\alpha^2 + (1 + c_{21}^2)\beta^2 - k_1^2,$$

$$a_{52} = c_{21}^2(\alpha^2 - \beta^2) + c_{31}^2\alpha\beta,$$

$$a_{57} = i\{\beta c_{21}^2 - (c_{31}^2 - c_{21}^2)\alpha\},$$

$$a_{61} = \alpha\beta c_{31}^2 - (\beta^2 - \alpha^2)c_{21}^2, \quad a_{64} = i\beta c_{41}^2,$$

$$a_{87} = \frac{T_0\omega(1 - \tau\omega)c_{41}^2}{c_{51}^2}, \quad a_{67} = -i\beta(c_{21}^2 + c_{31}^2),$$

$$a_{73} = \frac{\alpha^2 + \beta^2 - k_1^2}{c_{31}^2 + 2c_{21}^2}, \quad a_{75} = i\frac{c_{21}^2(\beta - \alpha) - \alpha c_{31}^2}{c_{31}^2 + 2c_{21}^2},$$

$$a_{76} = \frac{-i\beta(c_{21}^2 + c_{31}^2)}{c_{31}^2 + 2c_{21}^2}, \quad a_{78} = \frac{c_{41}^2}{c_{31}^2 + 2c_{21}^2},$$

$$a_{81} = \frac{T_0\omega\alpha c_{41}^2(1 - \tau\omega)}{c_{51}^2}, \quad a_{82} = \frac{T_0\omega\beta c_{41}^2(1 - \tau\omega)}{c_{51}^2},$$

$$a_{54} = i\alpha c_{41}^2,$$

$$a_{62} = -k_1^2 + \beta^2(c_{31}^2 + c_{21}^2) - \alpha^2(c_{21}^2 - 1),$$

$$a_{84} = (\alpha^2 + \beta^2) - i\frac{c_{e1}(1 - \tau\omega)\omega}{c_{51}^2}.$$

EIGENVAUE OF MATRIX DIFFERENTIAL EQUATION

We use eigenvalue approach to solve the differential equation (10). The characteristic equation of matrix, A is given by

$$-B_1\lambda^6 + B_2\lambda^4 - B_3\lambda^2 + B_4 = 0, \quad (13)$$

where

$$B_1 = a_{51} + a_{61} + a_{73} + a_{84} + a_{57}a_{75} + a_{67}a_{76},$$

$$\begin{aligned} B_2 &= a_{51}a_{62} - a_{52}a_{61} + a_{51}a_{73} + a_{62}a_{73} + a_{51}a_{84} - a_{54}a_{81} + a_{62}a_{84} - \\ & a_{82}a_{64} + a_{73}a_{84} - a_{54}a_{87}a_{75} + a_{51}a_{67}a_{76} - a_{52}a_{67}a_{75} + a_{87}a_{67}a_{75} \\ & + a_{87}a_{62}a_{78} + a_{57}a_{75}a_{84} - a_{57}a_{78}a_{81} + a_{67}a_{76}a_{84} - a_{78}a_{82}a_{67}, \end{aligned}$$

$$\begin{aligned} B_3 &= -a_{52}a_{61}a_{73} + a_{51}a_{62}a_{73} + a_{51}a_{62}a_{84} - a_{51}a_{62}a_{84} - a_{51}a_{64}a_{82} - \\ & a_{52}a_{61}a_{84} + a_{52}a_{64}a_{81} + a_{54}a_{61}a_{82} + a_{51}a_{73}a_{84} - a_{54}a_{73}a_{81} + a_{62}a_{73} \\ & a_{84} - a_{64}a_{73}a_{82} - a_{57}a_{61}a_{76}a_{84} + a_{57}a_{62}a_{75}a_{84} - a_{57}a_{64}a_{75}a_{82} \\ & + a_{57}a_{64}a_{76}a_{81} + a_{51}a_{67}a_{76}a_{84} - a_{52}a_{67}a_{75}a_{84} + a_{54}a_{67}a_{75}a_{82} \\ & - a_{54}a_{67}a_{76}a_{81} + a_{57}a_{61}a_{78}a_{82} - a_{57}a_{62}a_{78}a_{81} - a_{51}a_{67}a_{78}a_{82} \\ & + a_{52}a_{67}a_{78}a_{81} - a_{51}a_{87}a_{64}a_{76} + a_{52}a_{87}a_{64}a_{75} + a_{87}a_{54}a_{61}a_{75} - \\ & a_{87}a_{54}a_{62}a_{75} + a_{51}a_{87}a_{62}a_{78} - a_{52}a_{87}a_{61}a_{78}, \end{aligned}$$

$$B_4 = a_{51}a_{62}a_{73}a_{84} - a_{51}a_{64}a_{73}a_{82} - a_{52}a_{61}a_{73}a_{84} + a_{52}a_{64}a_{73}a_{81} + a_{54}a_{61}a_{73}a_{82} - a_{54}a_{62}a_{73}a_{81}.$$

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The roots of characteristic equation, $\pm\lambda_1, \pm\lambda_2, \pm\lambda_3$ and $\pm\lambda_4$ are eigenvalue of matrix A . Thus, the full form of displacement components and temperature field in three dimensions are given by

$$(u, v, w, \psi) = \sum_{n=1}^4 (1, b_n, d_n, f_n) A_n e^{i(\alpha x + \beta y - \omega t)} e^{-\lambda_n z} \quad (14)$$

where b_n, d_n and f_n are coupling constants and their expression are given below

$$b_n = \frac{S_n}{Q_n}, \quad d_n = \frac{R_n}{Q_n}, \quad f_n = \frac{P_n}{Q_n},$$

$$P_n = (c_{51}^2 a_{81} E_n - c_{51}^2 a_{67} a_{82} \lambda_n^2) (a_{67} D_n \lambda_n - a_{52} q_3)$$

$$- (\lambda_n c_{51}^2 q_3 a_{81} - q_4 \lambda_n a_{67}) (P_{12} a_{67} \lambda_n^2 + E_n a_{52}),$$

$$Q_n = -\lambda_n \{ (a_{54} a_{67} \lambda_n + c_{41}^2 a_{52}) (E_n c_{51}^2 a_{81} - c_{51}^2 a_{82} \lambda_n^2 a_{67})$$

$$+ (c_{51}^2 a_{81} - F_n a_{67}) (P_{12} a_{67} \lambda_n^2 + E_n a_{52}) \},$$

$$R_n = \frac{1}{a_0} \{ P_n \lambda_n (-c_{41}^2 a_{52} + a_{54} a_{67}) + Q_n (a_{67} D_n \lambda_n + a_{52} \lambda_n q_3) \},$$

$$a_0 = P_{12} a_{67} + E_n a_{52},$$

$$S_n = \frac{1}{a_{52}} \{ P_{12} \lambda_n R_n - a_{54} P_n - D_n Q_n \},$$

$$E_n = g_3 + (c_{31}^2 + 2c_{21}^2) \lambda_n - i(c_{21}^2 + c_{31}^2) \alpha \lambda_n,$$

$$q_3 = -i\beta c_{21}^2,$$

$$D_n = a_{51} + \lambda_n (\lambda_n - i c_{21} \beta), \quad P_{12} = i\alpha (c_{21}^2 - c_{31}^2),$$

$$g_3 = k_1^2 - (\alpha^2 + \beta^2), \quad q_4 = T_0 \omega \alpha c_{41}^2.$$

BOUNDARY CONDITIONS

The boundary surface at $z = 0$ is subjected to a time dependent thermal shock and the conditions are given by

$$\psi(x, y, 0, t) = F(t)H(a - |x|)(b - |y|), \quad (15)$$

$$\sigma_{zz}(x, y, 0, t) + \bar{\sigma}_{zz}(x, y, 0, t) = 0, \quad (16)$$

$$\sigma_{yz}(x, y, 0, t) + \bar{\sigma}_{yz}(x, y, 0, t) = 0, \quad (17)$$

$$\sigma_{xz}(x, y, 0, t) + \bar{\sigma}_{xz}(x, y, 0, t) = 0, \quad (18)$$

where H denotes Heaviside's function and $F(t) = \theta_0 e^{-\delta t}$ taking θ_0 as a constant.

Inserting the values of $\sigma_{ij}, \bar{\sigma}_{ij}$ and ψ into Eqs.(13)-(16), we get the following matrix equation

$$bA = c,$$

where

$$b_{1i} = f_i, \quad b_{2i} = i\lambda(\alpha + b_i\beta) - (\lambda + 1)d_i\lambda_i - \gamma\psi$$

$$+ H_0^2 \{ (b_i - 1)i(\beta - \alpha) - 2d_i\lambda_i \},$$

$$b_{3i} = d_i i\beta - \lambda_i b_i, \quad b_{4i} = -\lambda_i + d_i i\alpha,$$

$$c = [\theta_0 e^{-(d+\omega)t+i(\alpha x + \beta y)}, 0, 0, 0]^T, \quad A = [A_1, A_2, A_3, A_4]^T$$

Table 2: Values of the Parameters.

| Parameters | Values | Units | Parameters | Values | Units |
|------------|-----------------------|--|------------|-----------------------|----------------------------------|
| k_0 | 386 | NK ⁻¹ s ⁻¹ | α_i | 1.78×10^{-5} | K ⁻¹ |
| C_e | 383.1 | m ² K ⁻¹ s ⁻² | ρ | 8954 | Kgm ⁻³ |
| μ | 3.86×10^{10} | Nm ⁻² | λ | 7.76×10^{10} | Nm ⁻¹ s ⁻¹ |
| μ_0 | $4\pi \times 10^{-7}$ | Nms ² C ⁻² | H_0 | 10^3 | Cm ⁻¹ s ⁻¹ |
| τ | 0.02 | | T_0 | 293 | K |
| θ_0 | 10 | | δ | 0.1 | |
| x | 1 | | y | 2 | |
| α | 1.2 | | β | 1.4 | |
| t | 1 | | | | |

NUMERICAL DISCUSSION

We have developed a Matlab program for (u, v, w, ψ) and stress components. The effect of H_0 and τ on the propagation of waves have been examined. The following

relevant values of parameters for the generalized thermoelastic solid are considered Ezzat and Youssef (2005) in Table 2.

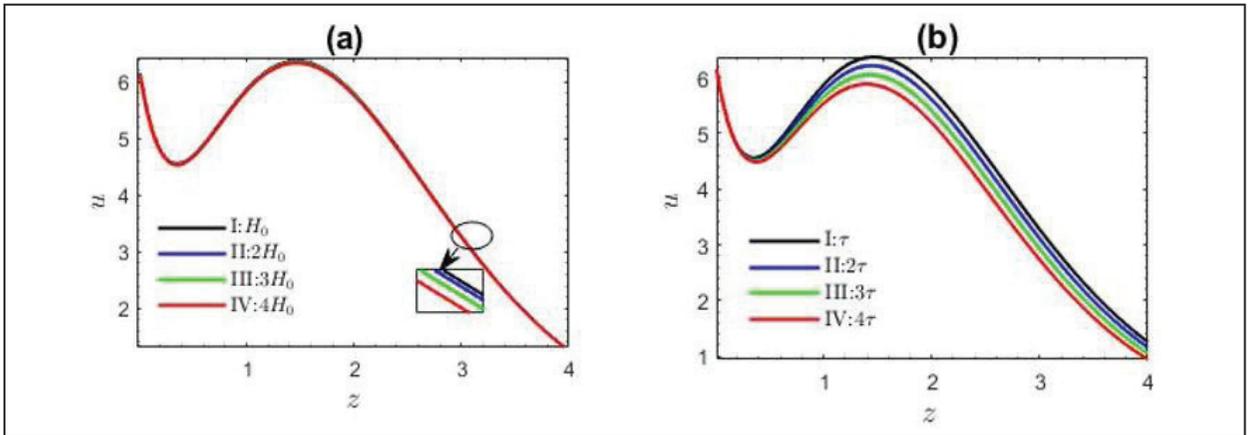


Fig. 1: Variation of u for Different Values of (a) H_0 and (b) τ .

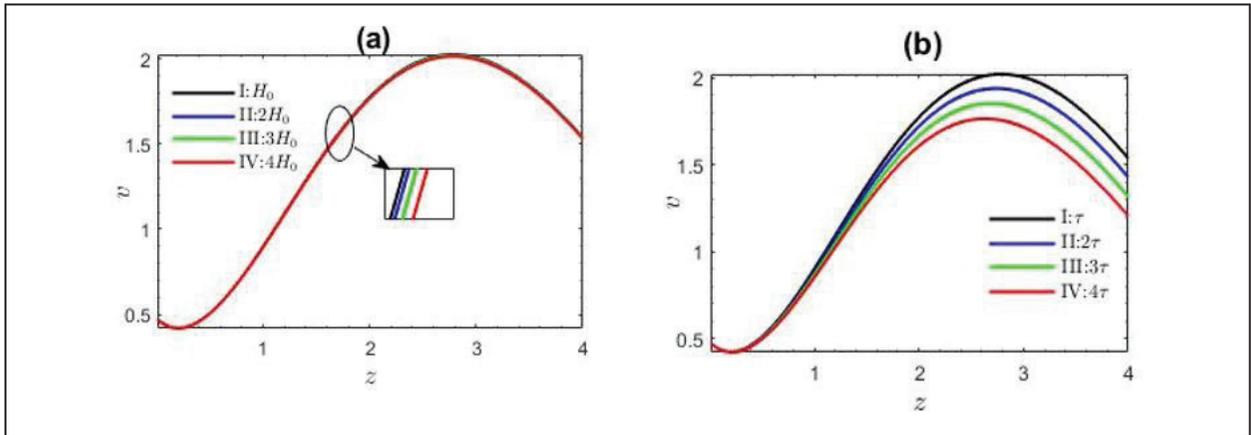


Fig. 2: Variation of v for Different Values of (a) H_0 and (b) τ .

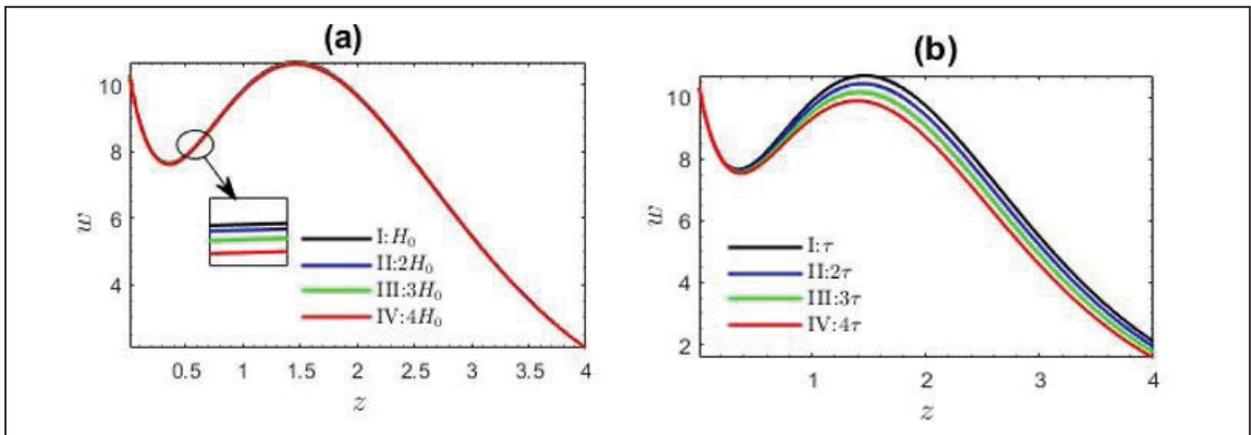


Fig. 3: Variation of w for Different Values of (a) H_0 and (b) τ .

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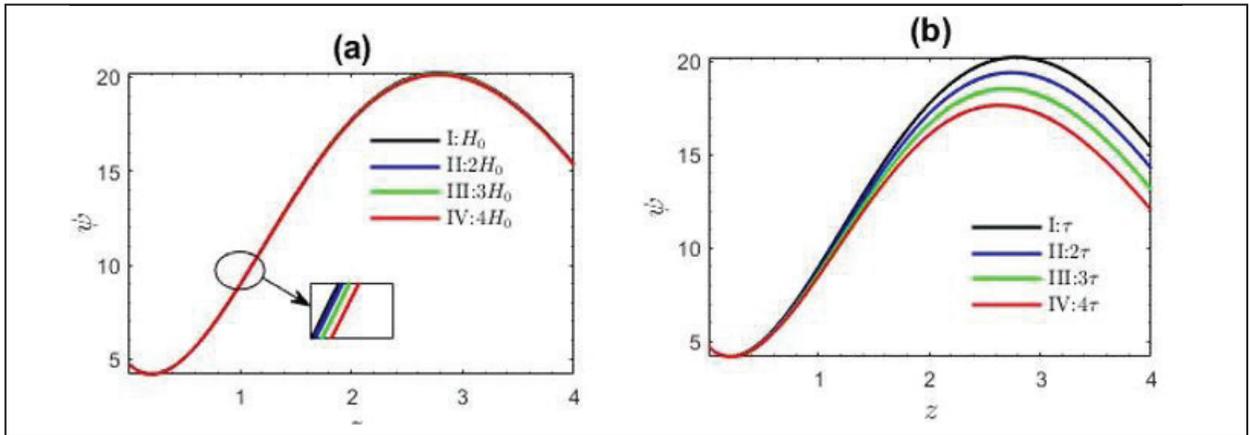


Fig. 4: Variation of ψ for Different Values of (a) H_0 and (b) τ .

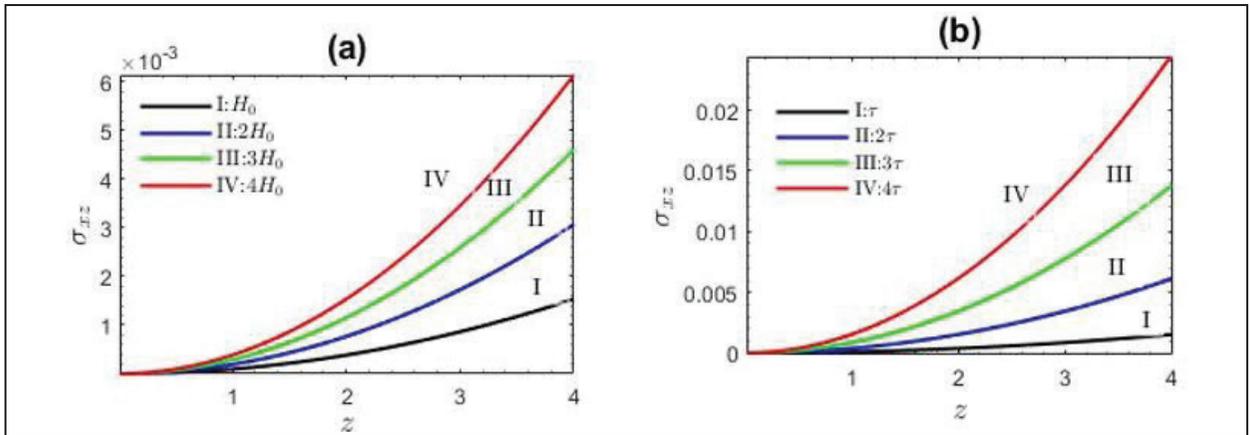


Fig. 5: Variation of σ_{xz} for Different Values of (a) H_0 and (b) τ .

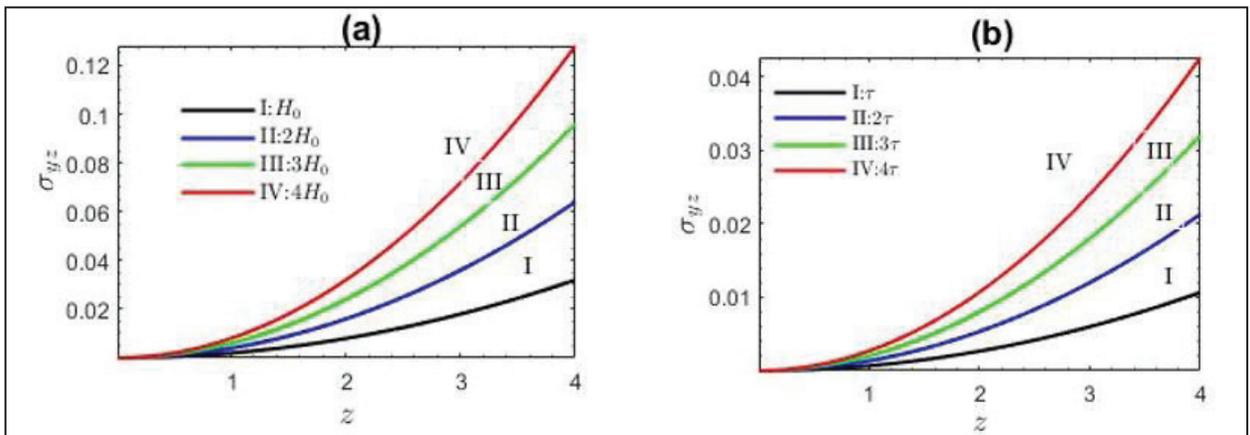


Fig. 6: Variation of σ_{yz} of Different Values of (a) H_0 and (b) τ .

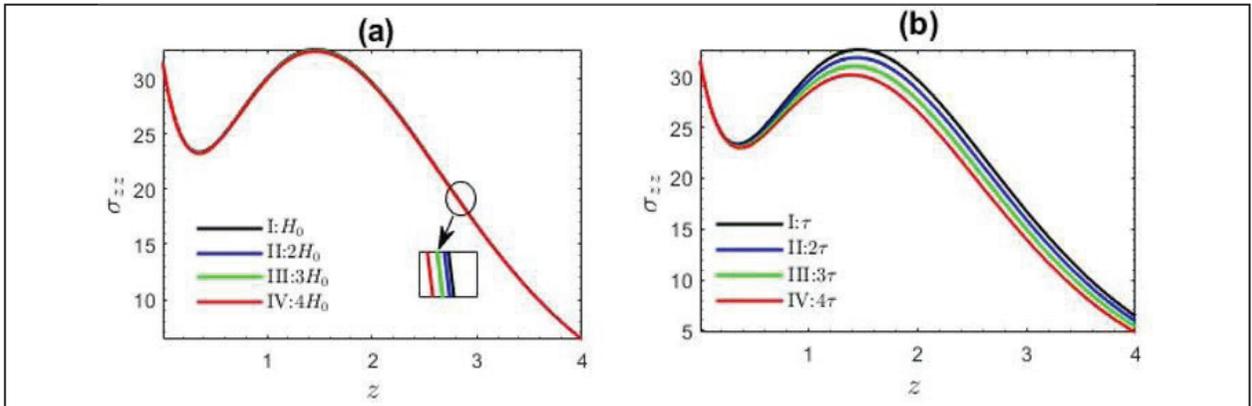


Fig. 7: Variation of σ_{zz} of Different Values of (a) H_0 and (b) τ .

Figures 1 to 4 represent the variation of the displacement components and temperature field with z for different values of H_0 and τ . We have seen a very interesting and beautiful effects of initial magnetic field and relaxation time on u, v, w . In Figure 1, the displacement component u begins from certain point which decreases with the enhancement of z upto $z = 0.29$ and then it increases and decreases with the increase of z . We have seen the effect of H_0 is smaller than τ . It is also clearly observed that the values of u decrease with the increase of H_0 and τ . Figure 2 shows that values of v increase upto $z = 2.67$ and then decrease with the increase of z . It is observed that the values of v decrease with the increase of H_0 and τ . Figure 3 shows similar nature of w with u . The variation of ψ with z are shown in Figure 4 which is similar with the nature of v . The variation of σ_{xz} and σ_{yz} with z are shown in Figures 5 and 6 respectively. It is observed that both these tensors increase with the increase of z at different values of H_0 and τ . The values of these tensors increase with the increase of H_0 and τ . Similar natures of σ_{zz} with z are observed in Figure 7. Thus, we have seen that all the effects of H_0 and τ to the displacement and stress are very small near origin.

CONCLUSION

We have analyzed the 3D problem of wave propagation in generalized magneto thermoelastic medium. The effects of H_0 and τ on displacement, stress tensors and temperature field are depicted graphically. We can conclude the following points:

- (i) The effect τ on the displacement and stress is very small near origin. The effect of H_0 on the components of displacement and temperature field is also very small
- (ii) The values of σ_{xz} and σ_{yz} increase with the increase of z .
- (iii) The values of σ_{xz} and σ_{yz} increase with the increase of H_0 and τ .

- (iv) Similar natures are observed for u, w and σ_{zz} .
- (v) Similar natures are observed for v and ψ .

REFERENCES

- Abbas IA (2012) Generalized magneto-thermoelastic interaction in a fiber-reinforced anisotropic hollow cylinder. *Int. J. Thermophys.* 33: 567-579.
- Abbas IA (2013) A GN model for thermoelastic interaction in an unbounded fiber-reinforced anisotropic medium with a circular hole. *Appl. Math. Lett.* 26(2): 232-239.
- Abbas IA, Kumar R, Chawla V (2012) Response of thermal source in a transversely isotropic thermoelastic half-space with mass diffusion by using a finite element method. *Chin. Phys. B.* 21(8): 084601.
- Abbas IA, Othman MI (2012) Plane waves in generalized thermo-microstretch elastic solid with thermal relaxation using finite element method. *Int. J. Thermophys.* 33: 2407-2423.
- Biot MA (1956) Thermoelasticity and irreversible thermodynamics. *J. Appl. Phys.* 27(3): 240-253.
- Dhaliwal RS, Sherief HH (1980) Generalized thermoelasticity for anisotropic media. *Q. Appl. Math.* 38(1): 1-8.
- Ezzat MA, Youssef HM (2005) Generalized magneto-thermoelasticity in a perfectly conducting medium. *Int. J. Solids Struct.* 42(24-25): 6319-6334.
- Lord HW, Shulman Y (1967) A generalized dynamical theory of thermoelasticity. *J. Mech. Physics.* 15(5): 299-309.
- Paria G (1966) Magneto-elasticity and magneto-thermoelasticity. *Adv. Appl. Mech.* 10: 73-112. Puri P (1973) Plane waves in generalized thermoelasticity. *Int. J. Engng.* 11(7): 735-744.
- Wang X, Dai HL (2004) Magnetothermodynamic stress and perturbation of magnetic field vector in an orthotropic thermoelastic cylinder. *Int. J. Eng. Sci.* 42(5-6): 539-556.
- Zorammuana C, Singh S (2016) Elastic waves in thermoelastic saturated porous medium. *Meccanica* 51(3): 593-609.